

#### **Hypothesis Testing**

#### Z Test

1. Company has established norms for the competency of Executives in an aptitude test. The historic population data suggests that average 73.2 with a standard deviation of 8.6. If 45 randomly selected persons and have an average 76.7, test the null hypothesis  $\mu$ = 73.2 against the alternative hypothesis  $\mu$ > 73.2 at the 0.01 level of significance

# One-Sample Z

## **Descriptive Statistics**

```
N Mean SE Mean 99% Cl for μ
            1.28 (73.40, 80.00)
 45 76.70
 μ: mean of Sample
 Known standard deviation = 8.6
Test
Null hypothesis
                    H_0: \mu = 73.2
Alternative hypothesis H₁: µ ≠ 73.2
Z-Value P-Value
    2.73
           0.006
```

2. Tests performed with a random sample of 40 diesel engines produced by a large manufacturer show that they have a mean thermal efficiency of 31.4% with a standard deviation of 1.6%. At the 0.01 level of significance, test the null hypothesis  $\mu = 32.3\%$ against the alternative hypothesis  $\mu \ll 32.3\%$ 

# One-Sample Z

Z-Value P-Value -3.56

0.000

## **Descriptive Statistics**

N Mean SE Mean 99% CI for 
$$\mu$$

40 31.400 0.253 (30.748, 32.052)

 $\mu$ : mean of Sample
Known standard deviation = 1.6

Test

Null hypothesis  $H_0$ :  $\mu$  = 32.3

Alternative hypothesis  $H_1$ :  $\mu \neq$  32.3



1t

- 3. The cycle time data of a process has been collected for 10 samples. The target cycle time is 180 sec. (File: Hypothesis Testing\_Practicedata.xls; Data Set 1)
  - Have we hit the target?

# One-Sample T: Cycle Time

## **Descriptive Statistics**

#### Test

```
Null hypothesis H_0: \mu = 180
Alternative hypothesis H_1: \mu \neq 180

T-Value P-Value

1.56 0.154
```

• If the results are that we cannot prove a difference, what is the power of the test to detect 1sec difference?

# **Power and Sample Size**

```
1-Sample t Test
Testing mean = null (versus ≠ null)
Calculating power for mean = null + difference
α = 0.05 Assumed standard deviation = 2.1
```

#### Results

|            | Sample |          |
|------------|--------|----------|
| Difference | Size   | Power    |
| 1          | 10     | 0.270673 |

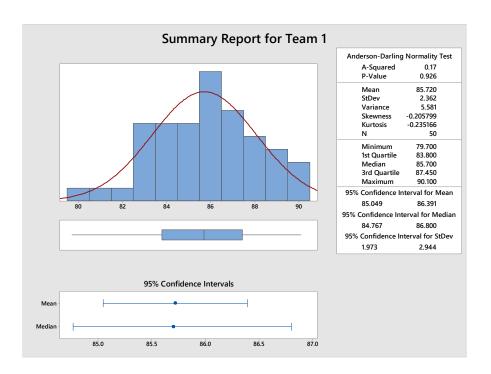
## Power Curve for 1-Sample t Test

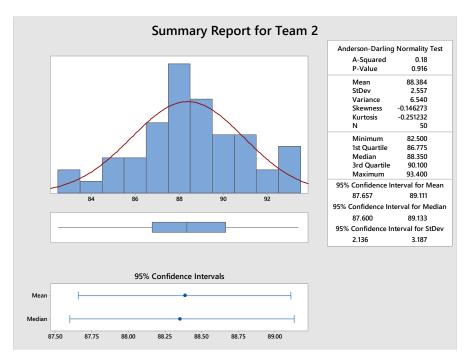


#### 2t

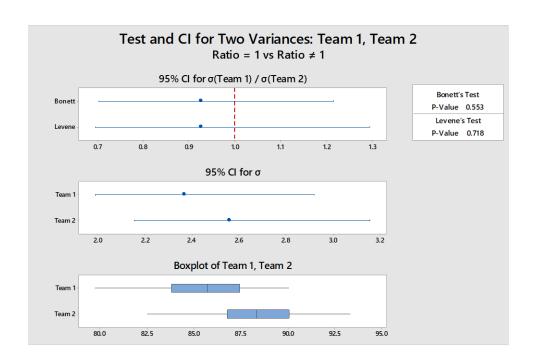
4. You are helping of your teams to improve their performance (Productivity %). You are comparing the data of these teams with few samples. Is there a difference between the team's performance. (File: Hypothesis Testing\_Practicedata.xls; Data Set 2)

Inference: Both data sets are normal









## Two-Sample T-Test and CI: Team 1, Team 2

### Method

 $\mu_1$ : mean of Team 1  $\mu_2$ : mean of Team 2 Difference:  $\mu_1 - \mu_2$ 

Equal variances are assumed for this analysis.

## **Descriptive Statistics**

| Sample | Ν  | Mean  | StDev | SE Mean |
|--------|----|-------|-------|---------|
| Team 1 | 50 | 85.72 | 2.36  | 0.33    |
| Team 2 | 50 | 88.38 | 2.56  | 0.36    |

## **Estimation for Difference**

|            | Pooled | 95% CI for       |
|------------|--------|------------------|
| Difference | StDev  | Difference       |
| -2.664     | 2.462  | (-3.641, -1.687) |

#### Test

Null hypothesis  $H_0: \mu_1 - \mu_2 = 0$ Alternative hypothesis  $H_1: \mu_1 - \mu_2 \neq 0$ 



#### Paired t

5. The weight of components from two different mold cavities are compared. Data is available to validate if there is any weight difference between cavities. (File: Hypothesis Testing\_Practicedata.xls; Data Set 3)

# Paired T-Test and CI: Cavity 1, Cavity 2

## **Descriptive Statistics**

| Sample   | Ν  | Mean  | StDev | SE Mean |
|----------|----|-------|-------|---------|
| Cavity 1 | 10 | 51.98 | 5.23  | 1.65    |
| Cavity 2 | 10 | 39.19 | 2.21  | 0.70    |

#### Estimation for Paired Difference

|       |       |         | 95% CI for    |
|-------|-------|---------|---------------|
| Mean  | StDev | SE Mean | μ_difference  |
| 12.79 | 5.11  | 1.62    | (9.13, 16.45) |

 $\mu_d$  difference: mean of (Cavity 1 - Cavity 2)

#### Test

 $\begin{array}{ll} \mbox{Null hypothesis} & \mbox{H}_0\colon \mu\_\mbox{difference} = 0 \\ \mbox{Alternative hypothesis} & \mbox{H}_1\colon \mu\_\mbox{difference} \neq 0 \\ \end{array}$ 

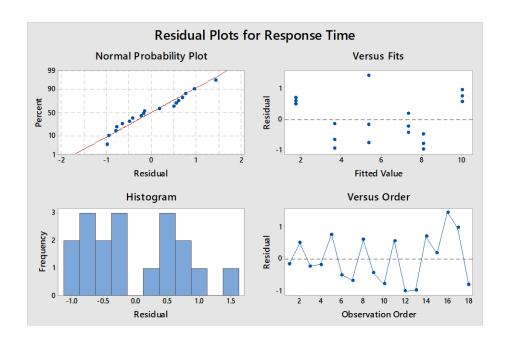
7.91 0.000

#### **ANOVA & GLM**

### ANOVA (B)

6. The data of response time for updating several files on different servers have been collected. As servers are of different configurations and the file sizes vary, you wish to know if their or both of them have an impact on response time. (File: Hypothesis Testing\_Practicedata.xls; Data Set 4)





## ANOVA: Response Time versus File in TB, Server

#### **Factor Information**

| Factor |            | Type Levels |   | Values   |  |
|--------|------------|-------------|---|----------|--|
|        | File in TB | Fixed       | 2 | 1.6, 1.9 |  |
|        | Server     | Fixed       | 3 | 4 8 12   |  |

## Analysis of Variance for Response Time

| Source     | DF | SS      | MS      | F     | Р     |
|------------|----|---------|---------|-------|-------|
| File in TB | 1  | 17.209  | 17.2089 | 26.69 | 0.000 |
| Server     | 2  | 123.143 | 61.5717 | 95.48 | 0.000 |
| Error      | 14 | 9.028   | 0.6448  |       |       |
| Total      | 17 | 149.380 |         |       |       |

## **Model Summary**

| S        | R-sq   | R-sq(adj) |
|----------|--------|-----------|
| 0.803020 | 93.96% | 92.66%    |

## **Residual Plots for Response Time**



7. The procurement team wants to use data to finalize their strategy to achieve the budget for the year. Procurement Managers have worked out 4 different approaches for few parts and estimated the potential saving per part. They haven't taken all parts as it is not practical for initial stage. Is there a distinct strategy evolving from this data? (File: Hypothesis Testing\_Practicedata.xls; Data Set 5)



## Mixed Effects Model: Saving per part versus Part\_1, ... rement Strategy

#### Method

Variance estimation Restricted maximum likelihood

DF for fixed effects Kenward-Roger

## **Factor Information**

| Factor               | Type   | Levels | Values  |
|----------------------|--------|--------|---|
| Part_1               | Random | 8      | A, B, C, D, E, F, G, H                              |
| Procurement Strategy | Fixed  | 4      | Alternate Sourcing, Redesign, Renegotiation, Vendor |

#### **Variance Components**

| Source | Var      | % of Total | SE Var   | Z-Value  | P-Value |
|--------|----------|------------|----------|----------|---------|
| Part_1 | 0.208122 | 18.71%     | 0.279692 | 0.744110 | 0.228   |
| Error  | 0.904219 | 81.29%     | 0.337217 | 2.681412 | 0.004   |
| Total  | 1.112341 |            |          |          |         |

-2 Log likelihood = 67,988086

#### **Tests of Fixed Effects**

| Term                 | DF Num | DF Den | F-Value | P-Value |
|----------------------|--------|--------|---------|---------|
| Procurement Strategy | 3.00   | 15.45  | 16.02   | 0.000   |

#### Model Summary

S R-sq R-sq(adj) 0.950904 77.13% 73.70%

#### Coefficients

| Term                 | Coef      | SE Coef  | DF    | T-Value   | P-Value |
|----------------------|-----------|----------|-------|-----------|---------|
| Constant             | 18.589432 | 0.272178 | 6.96  | 68.298848 | 0.000   |
| Procurement Strategy |           |          |       |           |         |
| Alternate Sourcing   | 1.792343  | 0.342736 | 14.77 | 5.229519  | 0.000   |
| Redesign             | 0.875676  | 0.342736 | 14.77 | 2.554960  | 0.022   |
| Renegotiation        | -1.303587 | 0.407283 | 15.84 | -3.200692 | 0.006   |

## Marginal Fits and Diagnostics for Unusual Observations

|     | Saving    |           |           |           |   |
|-----|-----------|-----------|-----------|-----------|---|
| Obs | per part  | Fit       | Resid     | Std Resid |   |
| 7   | 21.700000 | 19.465107 | 2.234893  | 2.319255  | R |
| 17  | 15.000000 | 17.225000 | -2.225000 | -2.255314 | R |

R Large residual

## **Conditional Fits and Diagnostics for Unusual Observations**

|     | Saving    |           |           |           |   |
|-----|-----------|-----------|-----------|-----------|---|
| Obs | per part  | Fit       | Resid     | Std Resid |   |
| 7   | 21.700000 | 19.937538 | 1.762462  | 2.166182  | R |
| 17  | 15.000000 | 16.846587 | -1.846587 | -2.218831 | R |

R Large residual

**Inference:** Mixed Effects Models was used as Part No has to be generalized for other parts not covered in this analysis. Parts are not significant however procurement strategy is.



8. Data of whether a team meets its daily production target or not is collected for 90 days along with few factors which are considered to have an association. Using chi-square tests, identify these factors. (File: Hypothesis Testing\_Practicedata.xls; Data Set 6)

## Chi-Square Test for Association: Daily Production Target ... jor Reasons

#### Rows: Daily Production Target (Met/No Columns: Line stopper Major Reasons

|                                    | Machine<br>Breakdown | Manpower<br>Issue | Material<br>Shortage | Quality<br>Issue | All |
|------------------------------------|----------------------|-------------------|----------------------|------------------|-----|
| Met                                | 5<br>5.778           | 18<br>6.644       | 1<br>7.511           | 2<br>6.067       | 26  |
| Not Met                            | 15<br>14.222         | 5<br>16.356       | 25<br>18.489         | 19<br>14.933     | 64  |
| All                                | 20                   | 23                | 26                   | 21               | 90  |
| Cell Contents<br>Count<br>Expected | count                |                   |                      |                  |     |

#### Chi-Square Test

|                  | Chi-Square | DF | P-Value |  |
|------------------|------------|----|---------|--|
| Pearson          | 39.209     | 3  | 0.000   |  |
| Likelihood Ratio | 39.943     | 3  | 0.000   |  |

## Chi-Square Test for Association: Daily Production Target ... h Changed

#### Rows: Daily Production Target (Met/No Columns: Sch Changed

|                        | N     | Υ     | Αll |
|------------------------|-------|-------|-----|
|                        |       |       |     |
| Met                    | 11    | 15    | 26  |
|                        | 8.67  | 17.33 |     |
|                        |       |       |     |
| Not Met                | 19    | 45    | 64  |
|                        | 21.33 | 42.67 |     |
| A.II                   | 20    |       |     |
| All                    | 30    | 60    | 90  |
| Cell Contents<br>Count |       |       |     |
| Expected               | count |       |     |
|                        |       |       |     |

#### **Chi-Square Test**

|                  | Chi-Square | DF | P-Value |
|------------------|------------|----|---------|
| Pearson          | 1.325      | 1  | 0.250   |
| Likelihood Ratio | 1.298      | 1  | 0.255   |



## Chi-Square Test for Association: Daily Production Target ... e-stoppers

## Rows: Daily Production Target (Met/No Columns: Line-stoppers

|                               |            | 0        | 1          | 2          | 3          | 4          | 5          | 6          | 7          | 8          | 9          | 10         |
|-------------------------------|------------|----------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| Met                           |            | 18<br>67 | 2<br>3.178 | 3<br>3.178 | 1<br>1.733 | 1<br>2.600 | 0<br>1.733 | 1<br>2.600 | 0<br>0.867 | 0<br>0.578 | 0<br>0.289 | 0          |
| Not Met                       | 21.3       | 12       | 9<br>7.822 | 8<br>7.822 | 5<br>4.267 | 8<br>6.400 | 6<br>4.267 | 8<br>6.400 | 3<br>2.133 | 2<br>1.422 | 1<br>0.711 | 1<br>0.711 |
| All                           |            | 30<br>Al | 11<br>I    | 11         | 6          | 9          | 6          | 9          | 3          | 2          | 1          | 1          |
| Met                           |            | 26       | _          |            |            |            |            |            |            |            |            |            |
| Not Met                       | 1<br>0.711 | 64       | ı          |            |            |            |            |            |            |            |            |            |
| All                           | 1          | 90       | )          |            |            |            |            |            |            |            |            |            |
| Cell Conte<br>Count<br>Expect |            |          |            |            |            |            |            |            |            |            |            |            |

#### **Chi-Square Test**

|                  | Chi-Square | DF |
|------------------|------------|----|
| Pearson          | 23.655     | 11 |
| Likelihood Ratio | 26.540     | 11 |

8 cell(s) with expected counts less than 1. Chi-Square approximation probably invalid. 18 cell(s) with expected counts less than 5.

## Chi-Square Test for Association: Daily Production Target ... t/Non-Met)

## Rows: Daily Production Target (Met/No Columns: First hour output (Met/Non-Met)

|                       | Met   | Not Met | Αll |
|-----------------------|-------|---------|-----|
|                       |       |         |     |
| Met                   | 21    | 5       | 26  |
|                       | 21.38 | 4.62    |     |
|                       |       |         |     |
| Not Met               | 53    | 11      | 64  |
|                       | 52.62 | 11.38   |     |
|                       |       |         |     |
| All                   | 74    | 16      | 90  |
| Cell Content<br>Count | s     |         |     |
| Expected              | count |         |     |
| Expetteu              | Count |         |     |

#### **Chi-Square Test**

|                  | Chi-Square | DF | P-Value |
|------------------|------------|----|---------|
| Pearson          | 0.053      | 1  | 0.818   |
| Likelihood Ratio | 0.052      | 1  | 0.819   |

1 cell(s) with expected counts less than 5.



## Chi-Square Test for Association: Daily Production Target ... el Changes

## Rows: Daily Production Target (Met/No Columns: No. of Model Changes

|         | 3     | 4     | 5     | 6      | 7      | 8     | 9     | 10    | 11    | ΑII |
|---------|-------|-------|-------|--------|--------|-------|-------|-------|-------|-----|
|         |       |       |       |        |        |       |       |       |       |     |
| Met     | 2     | 9     | 5     | 9      | 1      | 0     | 0     | 0     | 0     | 26  |
|         | 0.578 | 3.178 | 3.178 | 7.222  | 5.489  | 3.178 | 1.444 | 0.578 | 1.156 |     |
|         |       |       |       |        |        |       |       |       |       |     |
| Not Met | 0     | 2     | 6     | 16     | 18     | 11    | 5     | 2     | 4     | 64  |
|         | 1.422 | 7.822 | 7.822 | 17.778 | 13.511 | 7.822 | 3.556 | 1.422 | 2.844 |     |
|         |       |       |       |        |        |       |       |       |       |     |
| All     | 2     | 11    | 11    | 25     | 19     | 11    | 5     | 2     | 4     | 90  |

Cell Contents Count Expected count

#### **Chi-Square Test**

|                  | Chi-Square | DF |
|------------------|------------|----|
| Pearson          | 36.109     | 8  |
| Likelihood Ratio | 42.112     | 8  |

2 cell(s) with expected counts less than 1. Chi-Square approximation probably invalid. 11 cell(s) with expected counts less than 5.

# Chi-Square Test for Association: Daily Production Target ... Month End

## Rows: Daily Production Target (Met/No Columns: Month End

|                        | Month End   | Not Month<br>End | All |
|------------------------|-------------|------------------|-----|
| Met                    | 23          | 3                | 26  |
|                        | 12.71       | 13.29            |     |
| Not Met                | 21<br>31.29 | 43<br>32.71      | 64  |
| All                    | 44          | 46               | 90  |
| Cell Contents<br>Count |             |                  |     |

### **Chi-Square Test**

Expected count

|                  | Chi-Square | DF | P-Value |  |
|------------------|------------|----|---------|--|
| Pearson          | 22.914     | 1  | 0.000   |  |
| Likelihood Ratio | 25.122     | 1  | 0.000   |  |



## Chi-Square Test for Association: Daily Production Target ... ay of Week

## Rows: Daily Production Target (Met/No Columns: Day of Week

|               | Fri    | Mon   | Sat   | Thur   | Tue    | Wed    | Αll |
|---------------|--------|-------|-------|--------|--------|--------|-----|
|               |        |       |       |        |        |        |     |
| Met           | 1      | 2     | 0     | 6      | 12     | 5      | 26  |
|               | 6.644  | 0.867 | 2.022 | 6.644  | 4.333  | 5.489  |     |
| Not Met       | 22     | 1     | 7     | 17     | 3      | 14     | 64  |
|               | 16.356 | 2.133 | 4.978 | 16.356 | 10.667 | 13.511 |     |
| All           | 23     | 3     | 7     | 23     | 15     | 19     | 90  |
| Cell Contents |        |       |       |        |        |        |     |

Count Expected count

#### **Chi-Square Test**

|                  | Chi-Square | DF |
|------------------|------------|----|
| Pearson          | 30.894     | 5  |
| Likelihood Ratio | 32.847     | 5  |

1 cell(s) with expected counts less than 1.

Chi-Square approximation probably invalid. 5 cell(s) with expected counts less than 5.

## Chi-Square Test for Association: Daily Production Target ... t/No, Shift

## Rows: Daily Production Target (Met/No Columns: Shift

|              | Α      | В      | С     | ΑII |
|--------------|--------|--------|-------|-----|
|              |        |        |       |     |
| Met          | 17     | 5      | 4     | 26  |
|              | 14.156 | 8.667  | 3.178 |     |
|              |        |        |       |     |
| Not Met      | 32     | 25     | 7     | 64  |
|              | 34.844 | 21.333 | 7.822 |     |
|              |        |        |       |     |
| All          | 49     | 30     | 11    | 90  |
| Cell Content | _      |        |       |     |
| Count        | 2      |        |       |     |
| Expected     | count  |        |       |     |

### **Chi-Square Test**

|                  | Chi-Square | DF | P-Value |
|------------------|------------|----|---------|
| Pearson          | 3.284      | 2  | 0.194   |
| Likelihood Ratio | 3.491      | 2  | 0.175   |

1 cell(s) with expected counts less than 5.



## Chi-Square Test for Association: Daily Production Target ... rst Half & 2

## Rows: Daily Production Target (Met/No Columns: Time Window (1 - First Half & 2

|         | A1           | A2           | B1    | B2    | C1    | C2    | ΑII |
|---------|--------------|--------------|-------|-------|-------|-------|-----|
| Met     | 15           | 2            | 3     | 2     | 3     | 1     | 26  |
|         | 8.667        | 5.489        | 7.511 | 1.156 | 2.889 | 0.289 |     |
| Not Met | 15<br>21.333 | 17<br>13.511 |       |       |       |       | 64  |
| All     | 30           | 19           | 26    | 4     | 10    | 1     | 90  |

Cell Contents Count Expected count

## **Chi-Square Test**

|                  | Chi-Square | DF |
|------------------|------------|----|
| Pearson          | 16.772     | 5  |
| Likelihood Ratio | 17.473     | 5  |

2 cell(s) with expected counts less than 1. Chi-Square approximation probably invalid. 5 cell(s) with expected counts less than 5.